



**80** Pages  
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# EXERCISE BOOK CAHIER D'EXERCICES

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SUBJECT/SUJET ELEC 202



ASSEMBLED IN CANADA WITH IMPORTED MATERIALS  
ASSEMBLÉ AU CANADA AVEC DES MATIÈRES IMPORTÉES

12107

# ELEC 202 LEC 1

1/02/19

LEC

Ohm's Law  $GV = I$  where  $G = \frac{1}{R}$  (conductance)  
\* in Siemens

Complex numbers

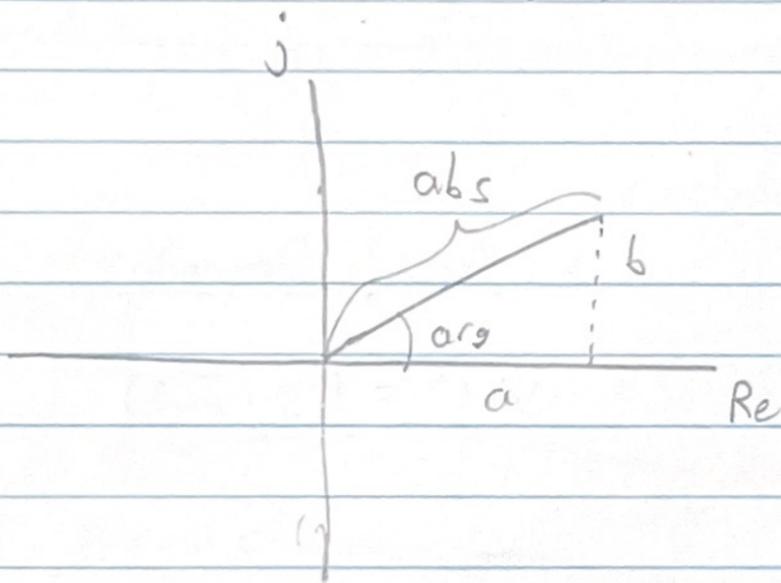
$$(3 + j4)(5 - j7) = 43 - j$$

Rectangular form:

$$(3, 4)(5, -7) = (43, -1)$$

Polar Form:

abs | arg°



$$v(t) = 30 \cos(377t + 25^\circ)$$

$$\omega = 2\pi f$$

$$\underline{v} = 30 \angle 25^\circ$$

# Laplace Domain

1/07/18  
LEC

## Laplace

$$f(t) \longleftrightarrow F(s)$$

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$e^{-at} u(t) \longleftrightarrow \frac{1}{s+a}$$

$$t u(t) \longleftrightarrow \frac{1}{s^2}$$

$$\delta(t) \longleftrightarrow 1$$

## For the inductor

Time Domain

$$V(t) = L \frac{di}{dt}$$

Laplace Domain

$$V(s) = \underline{Ls} \cdot I(s)$$

Impedance (Z) of the inductor  
in Ohms

## For the Capacitor

Time Domain

$$I(t) = C \frac{dV}{dt}$$

Laplace Domain

$$I(s) = Cs \cdot V(s)$$

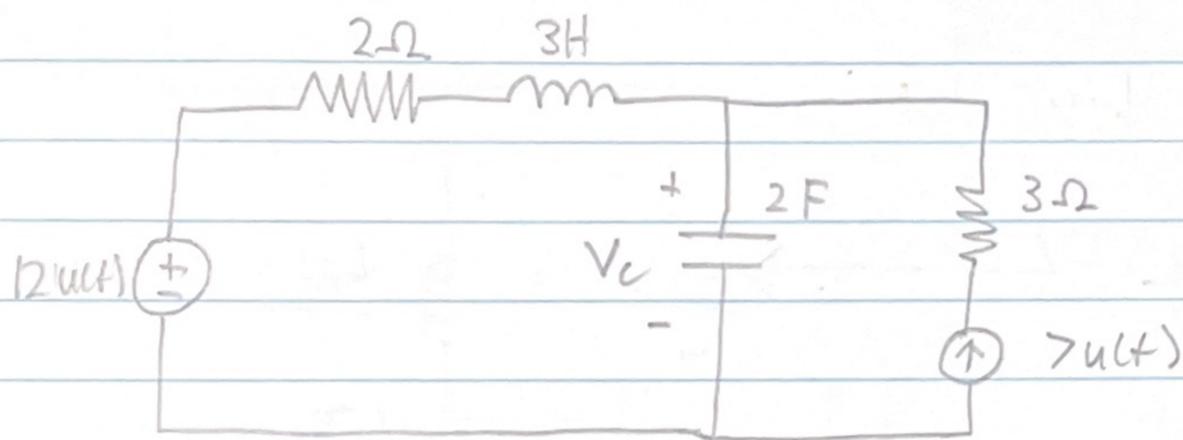
$$V(s) = \underline{\frac{1}{Cs}} \cdot I(s)$$

Impedance (Z) of the  
Capacitor in Ohms

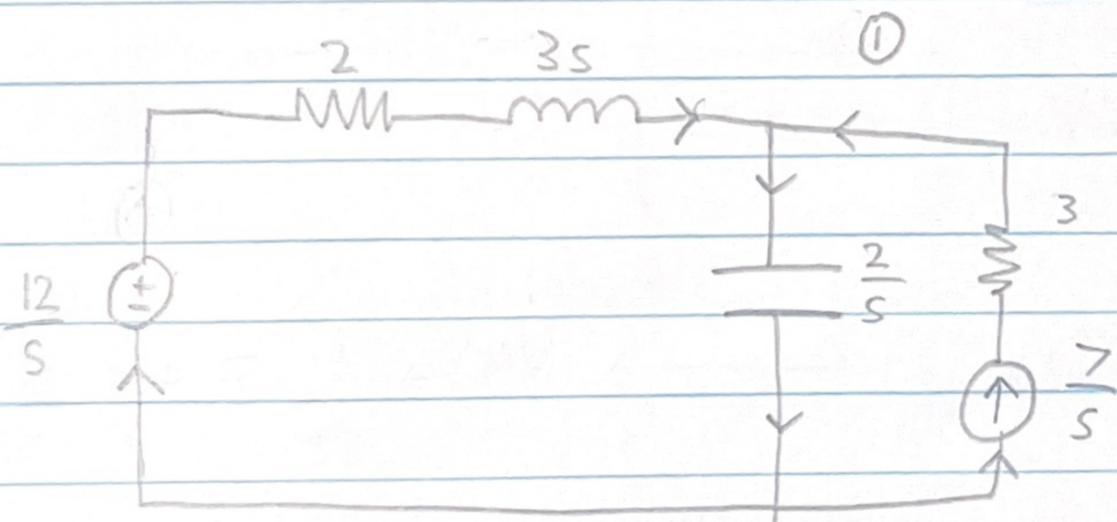
## MNA in Laplace Domain



$$I(s) = V_3 - V_7$$



Laplace Domain



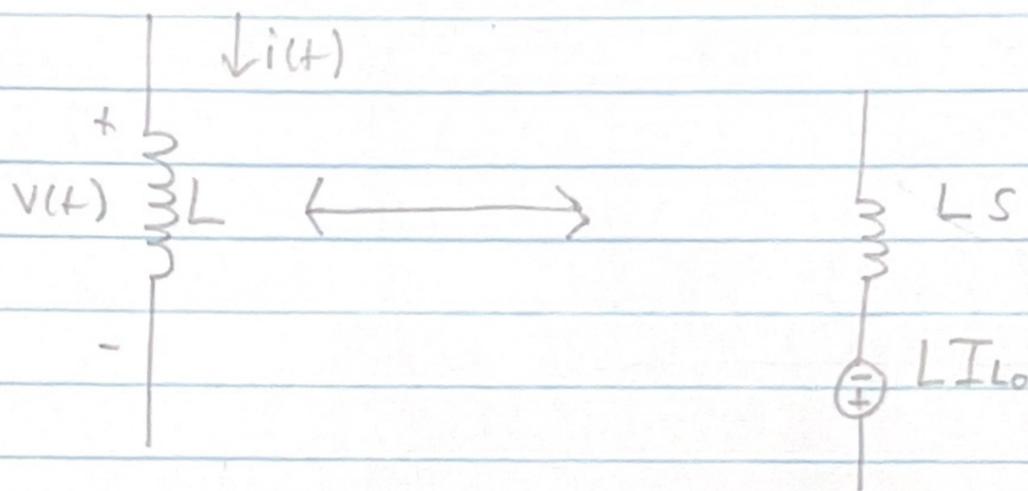
$$\text{KCL}_1: \frac{7}{s} + \frac{\frac{12}{s} - V_1}{2 + 3s} = \frac{V_1 \cdot s}{2}$$

$$V_1 = \frac{4 \cdot 2s + 52}{3s^3 + 2s^2 + 2s}$$

## Accounting For IC

### Inductors:

$$V(t) = L \frac{di}{dt} \longleftrightarrow V(s) = Ls I(s) - L I_{L0}$$



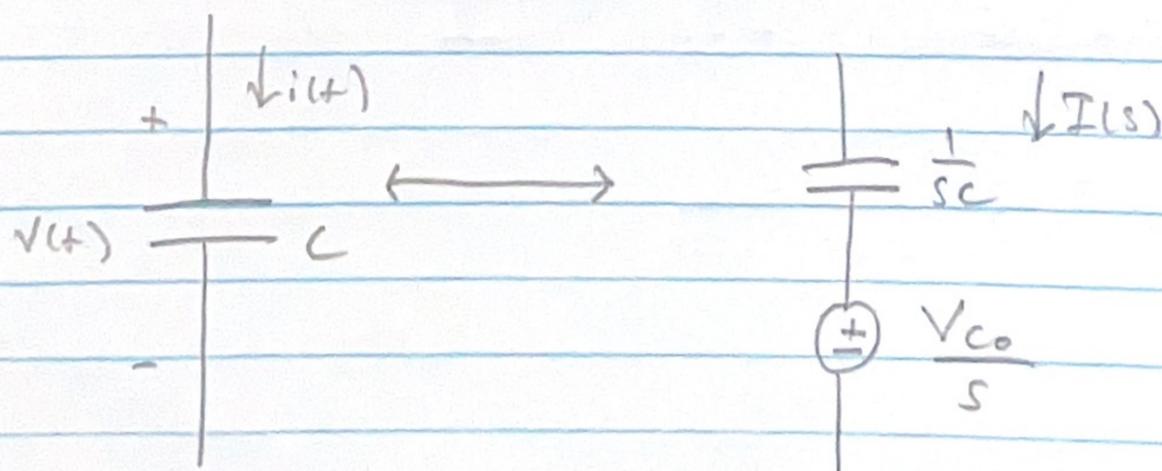
### Capacitors:

$$i(t) = C \frac{dV}{dt}$$

$$V(t) = \frac{1}{C} \int i(t) dt \longleftrightarrow V(s) = \frac{1}{sC} I + \frac{1}{C} \frac{q_0}{s}$$

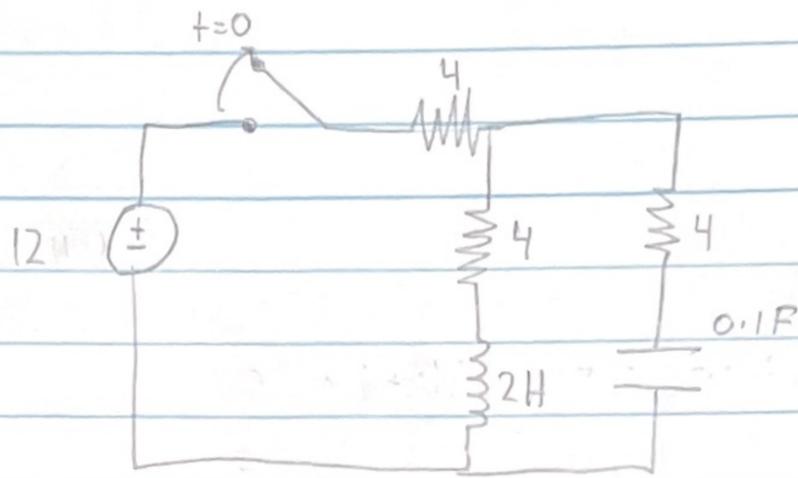
*initial voltage*

$$V(s) = \frac{1}{sC} I + \frac{V_{C0}}{s}$$



# Laplace Transform

1/09/19  
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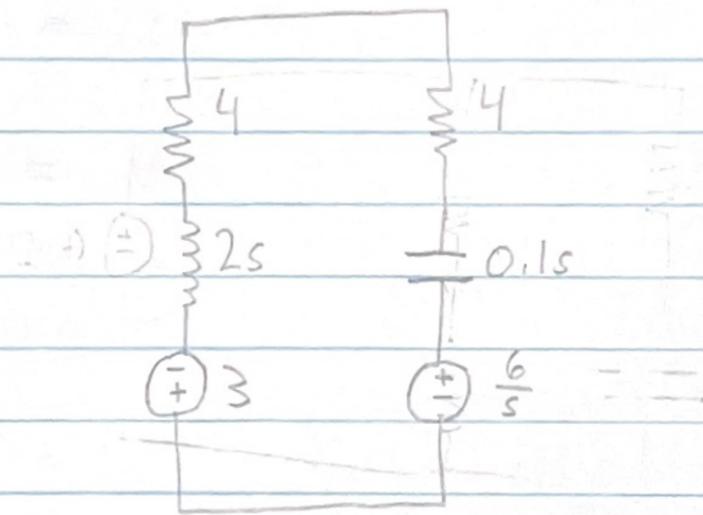
DC SS Findings:

$$I_L = 1.5A$$

$$V_C = 6V$$

$t > 0$

Laplace Domain

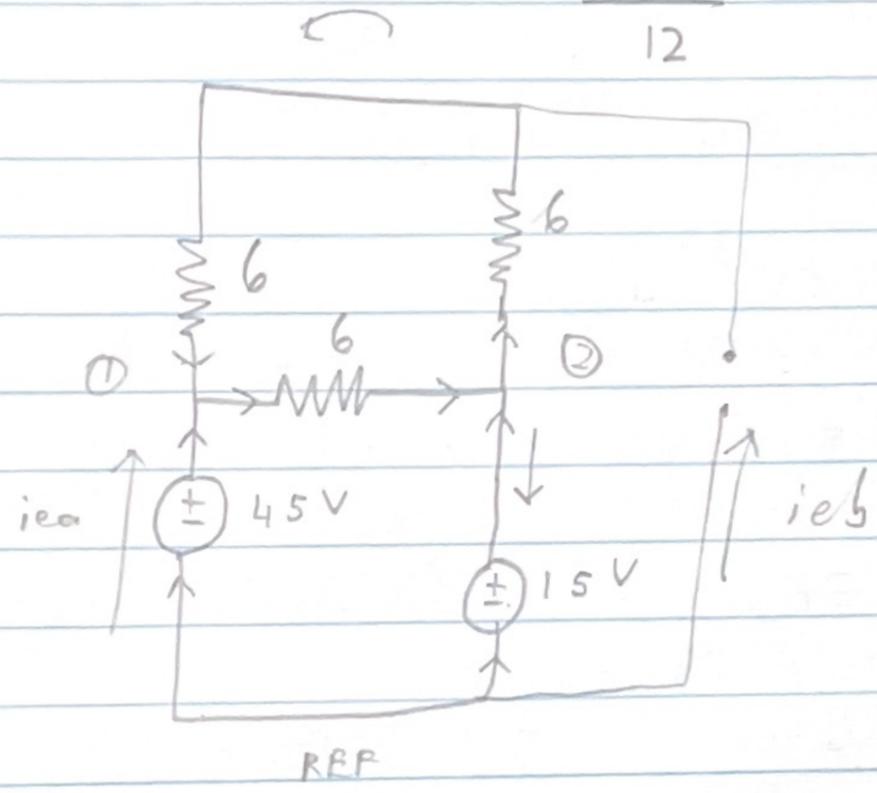


$$I_L = \frac{3 + \frac{6}{s}}{4 + 4 + 2s + 0.1s} = \frac{3}{2} \cos(t) e^{-2t}$$

2nd Order Circuits

# of reactive elements (capacitors/inductors) gives order

$$I = \frac{15 - 45}{12} \quad V_c = 30V$$



$$V_1: i_{ea} + \frac{V_2 - V_1}{12} = \frac{V_1 - V_2}{6}$$

$$V_2: \frac{V_1 - V_2}{6} + i_{eb} = \frac{V_2 - V_1}{12}$$

$$EVL_1: 0 + 45 = V_1$$

$$EVL_2: 0 + 15 = V_2$$

$$V_2: \frac{45 - 15}{6} + i_{eb} = \frac{15 - 45}{12}$$

$$\therefore V_1 =$$

$$V_2 =$$

EVL!

$V_2:$

$$V_2: \frac{45 - 15}{6} = \frac{45 - 15}{12}$$

# Frequency Response and Bode Plots

## AC steady state

$$v(t) = 9 \cos(300t + 45^\circ) \longleftrightarrow \bar{v} = \frac{9}{\sqrt{2}} \angle 45^\circ$$

$$\begin{aligned} \bar{s} &= p + jq \\ &= \bar{v} \cdot \bar{i} \\ &= s \angle \theta \end{aligned}$$

## Transfer Function



$$Y(s) = H(s) X(s)$$

\* kill internal sources, kill I.C.s  $\rightarrow$  Find  $\frac{Y(s)}{X(s)} = H(s)$

## Decibels from Transfer Function ( $\alpha_{dB}$ )

$$\alpha_{dB} = 20 \log_{10} |H(s)| \Big|_{s=j\omega}$$

$$H(s) = \frac{a_n s (s+z_1)(s+z_2) \dots (s+z_n)}{(s+p_1)(s+p_2) \dots (s+p_n)}$$

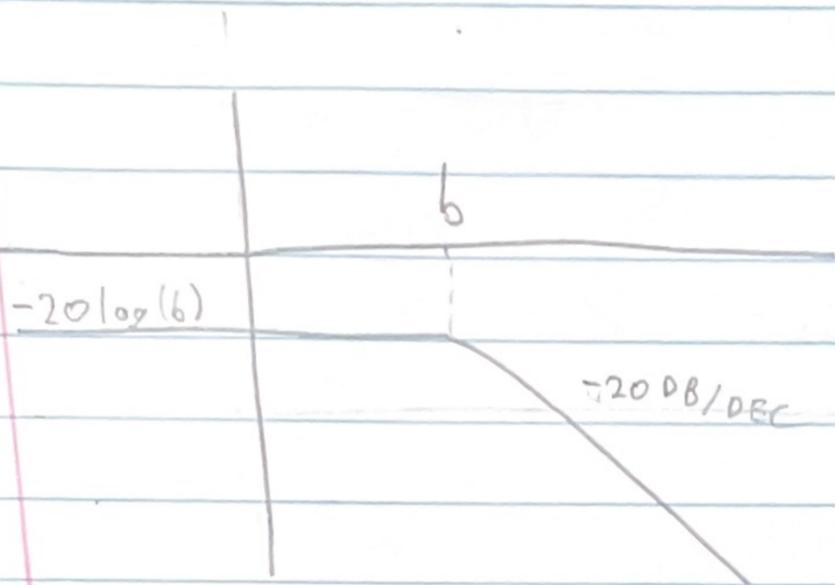
zeros

poles

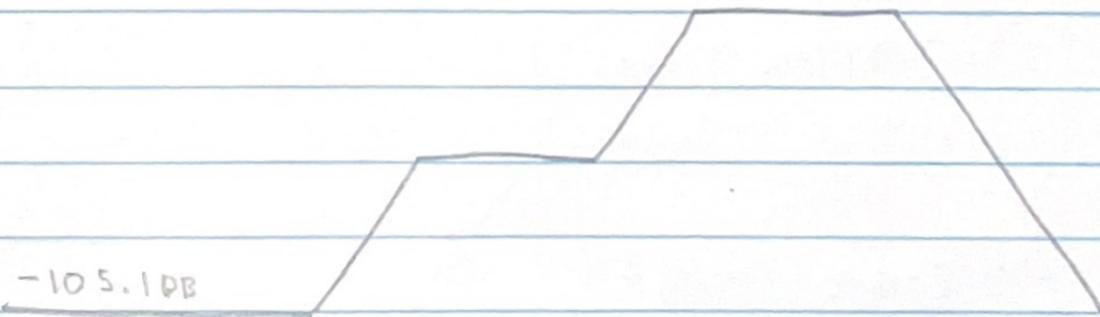


# Bode Plots

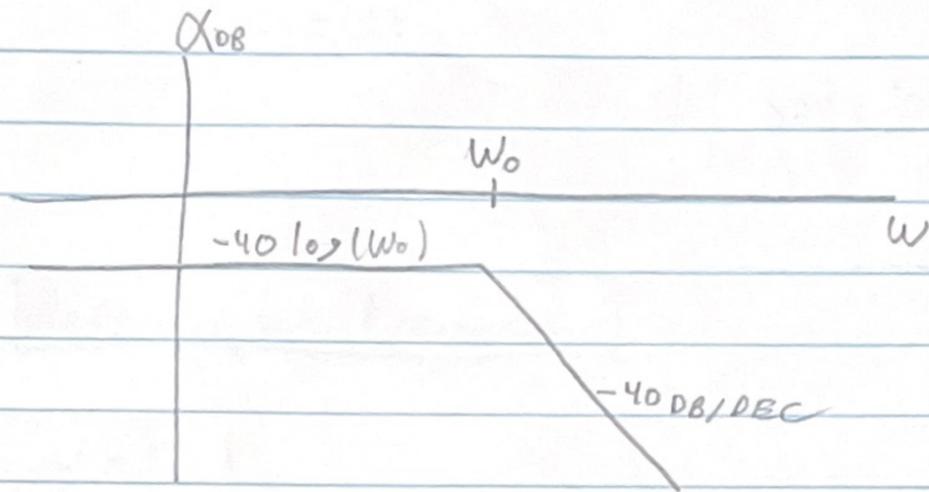
$$H(s) = \frac{1}{(s+b)}$$



$$H(s) = \frac{(s+20)(s+500)}{(s+150)(s+1200)(s+10,000)}$$



$$H(s) = \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$



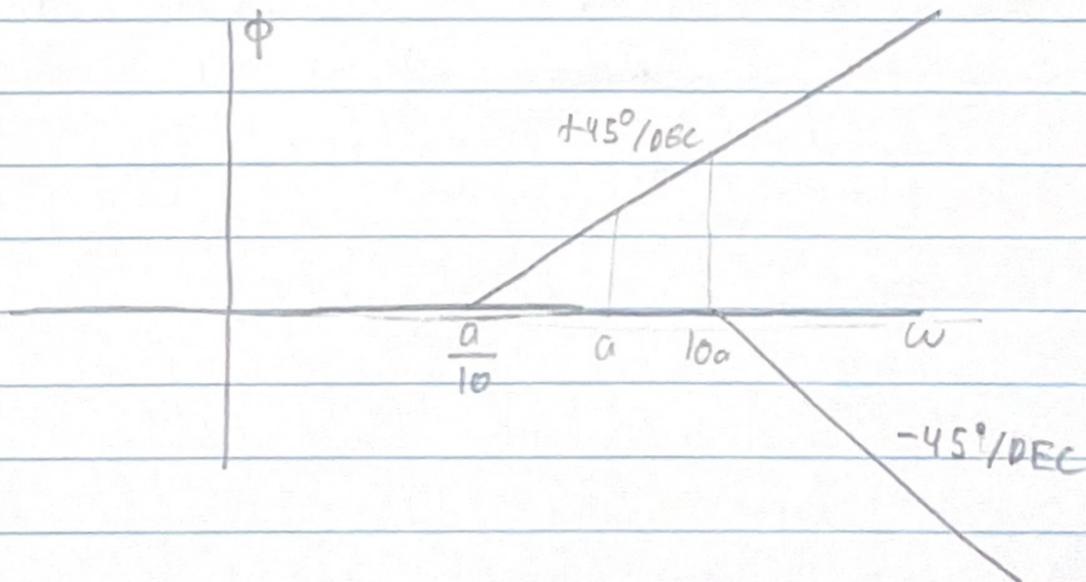
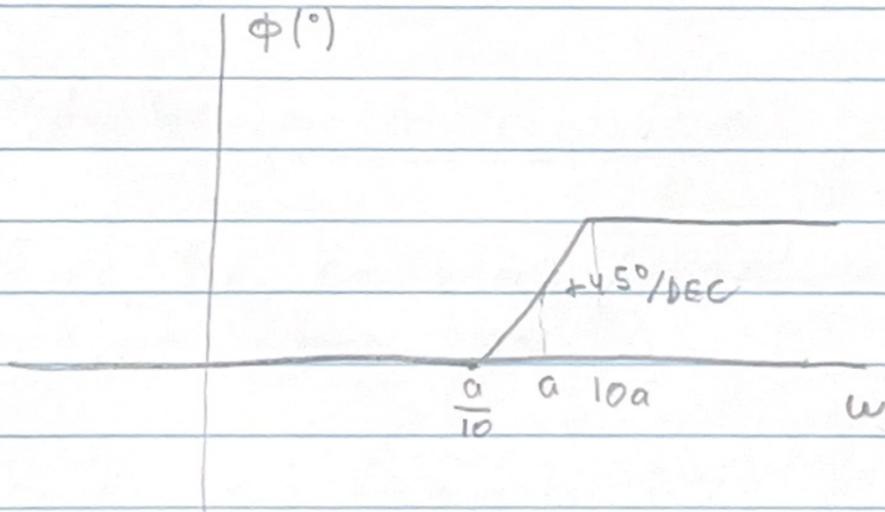
Phase

$$H(s) = j\omega \text{ (shift } x\text{-axis up } 90^\circ)$$

$$H(s) = 5s(s+a)$$

→ no effect

$$H(s) = (s+a)$$



## Resonance

1/28/19  
LEC

$$H(s) = \frac{Y(s)}{X(s)} \begin{cases} \alpha |_{s=j\omega} \text{ in dB} \\ \phi |_{s=j\omega} \text{ in } ^\circ \end{cases}$$

## Resonance Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

## Quality Factor

$$Q = \frac{\omega_0}{BW}$$

\* Another way to see it:

Find the equivalent impedance  $Z$  of the circuit, where  $s = j\omega$

Find the frequency ( $\omega$ ) where the angle of the impedance is zero, i.e.  $\text{Im}(Z(\omega)) = 0$  then that  $\omega$  is the resonance frequency

# Filters

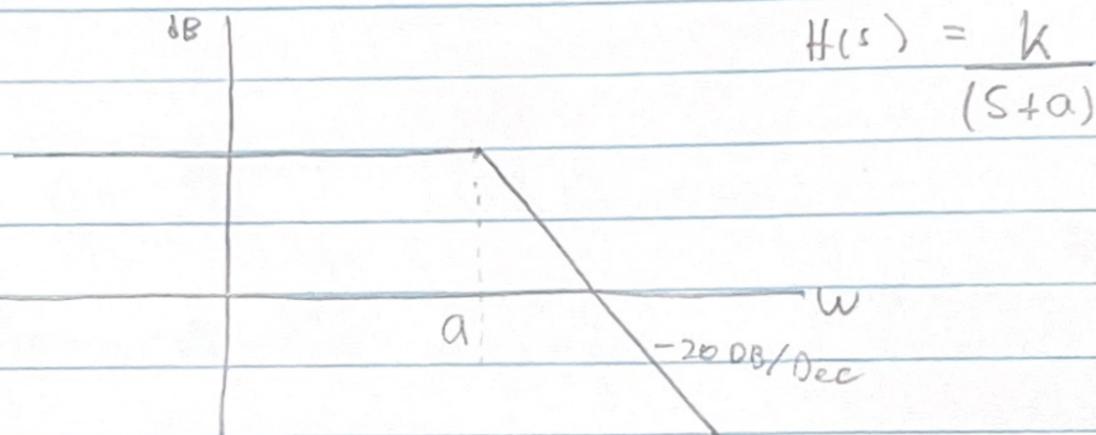
1/30/19

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Pass band  $\rightarrow \omega_p = \text{cutoff frequency}$

Filters  $\rightarrow 4$  types

\* Edges of the filter are the poles of the function

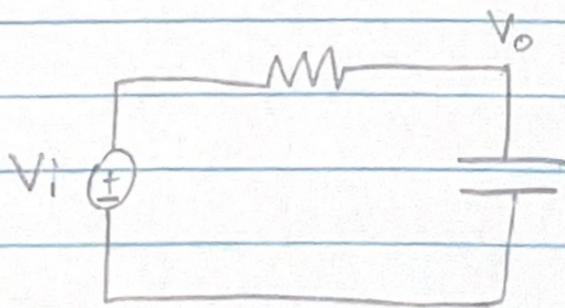


Approximated  $\omega_c = \text{Pole}$   
( $\omega_{\text{cutoff}}$ )

$$\frac{3s - 1}{10,000}$$

$$H(s) = \frac{25000}{s + 6500}$$

$$A_{PB} = \frac{25000}{\sqrt{(10^4)^2 + 6500^2}} \rightarrow \text{calculator}$$



$$\text{KVL: } 0 + V_i - V_o = V_o \cdot C \cdot s$$

$$\frac{V_o}{V_i} =$$

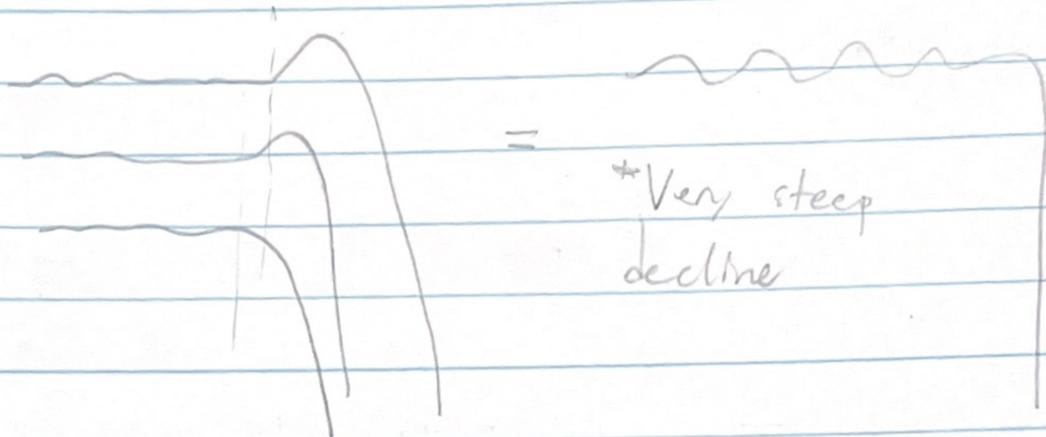
# Filters

2/4/19

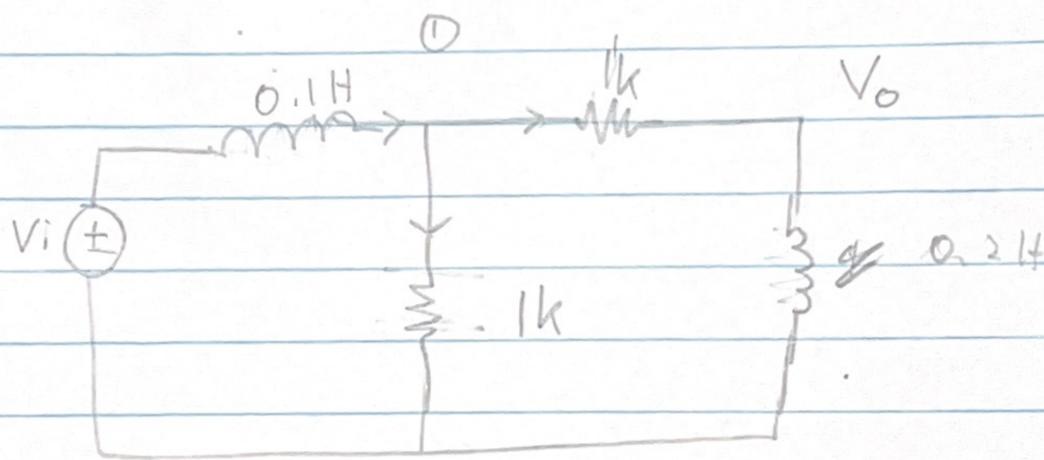
LEC

## Butterworth Filters (Chebyshev \*\*)

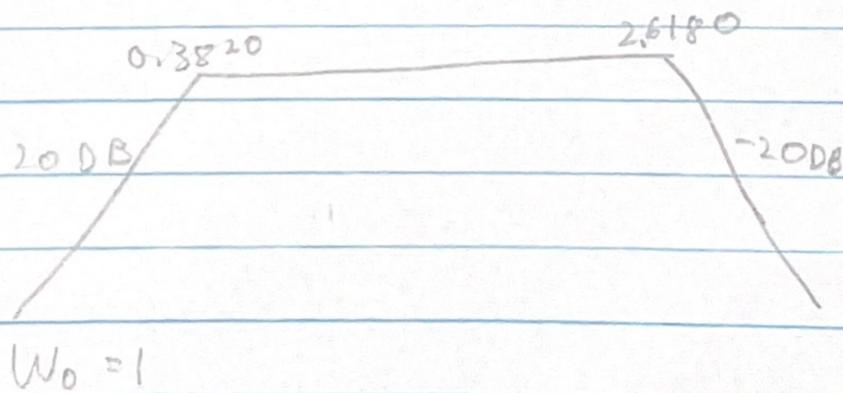
Damplage Factor  
↓



\*Very steep decline



$$H(s) = \frac{s}{s^2 + 3s + 1}$$



~~Q = 0.707~~

$$\text{KCL}_1: \frac{V_i - V_1}{0.1s} = \frac{V_1}{1k} + \frac{V_1 - V_o}{1k}$$

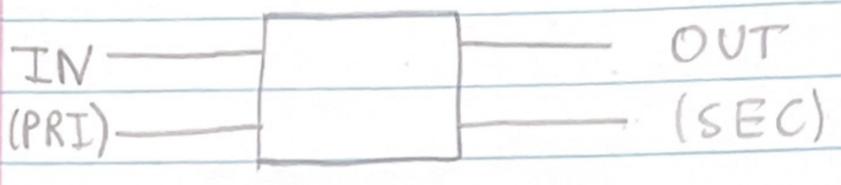
$$BW = \omega_2 - \omega_1$$

$$\text{EQ}_2: \frac{V_1 - V_o}{1k} = \frac{V_o - 0}{s \cdot 0.2}$$

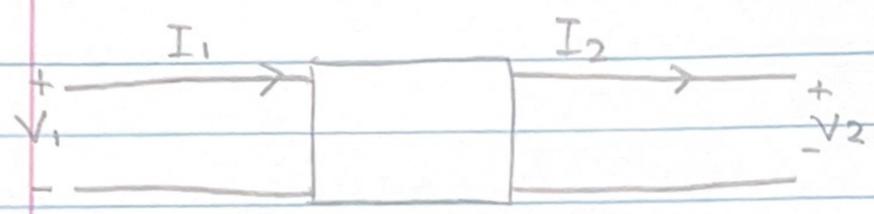
# Two-Port Networks

2/6/19  
LEC

## Ideal Transformer



Step down

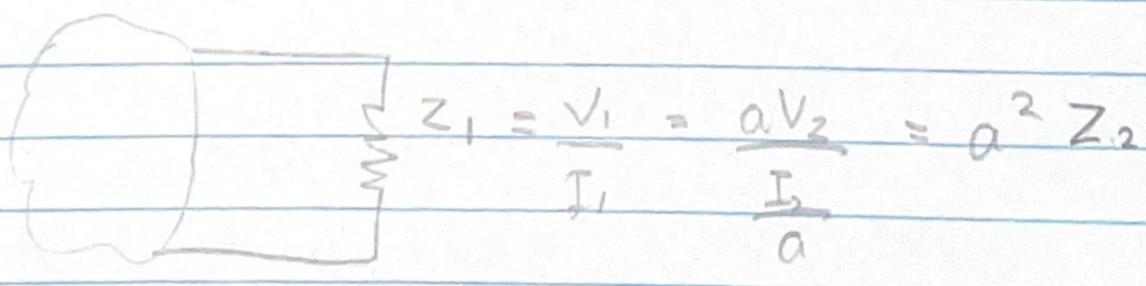
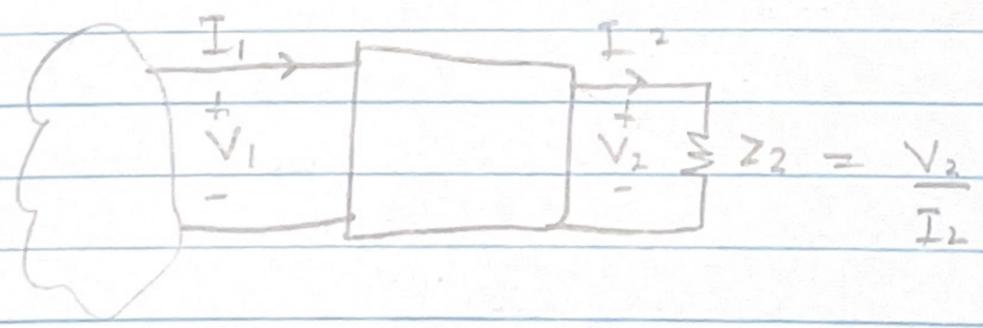


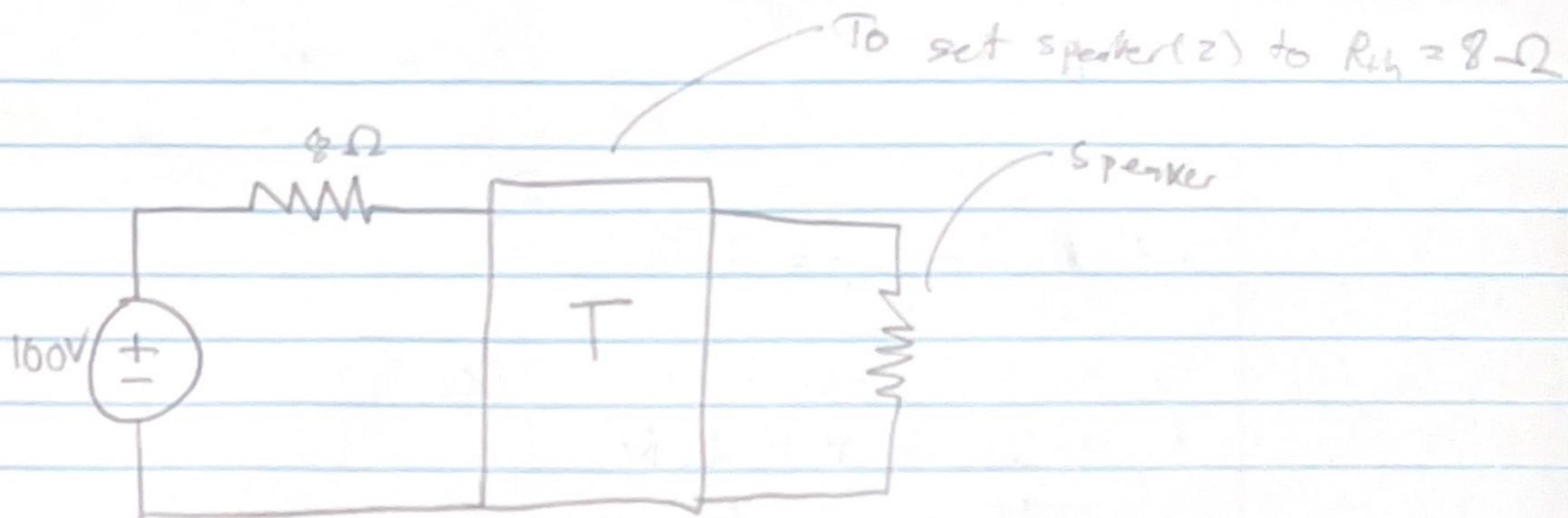
$$V_1 = a V_2$$

Transformation ratio

$$I_2 = a I_1$$

$$P_1 = V_1 I_1 = a V_2 \frac{I_2}{a} = V_2 I_2 = P_2$$

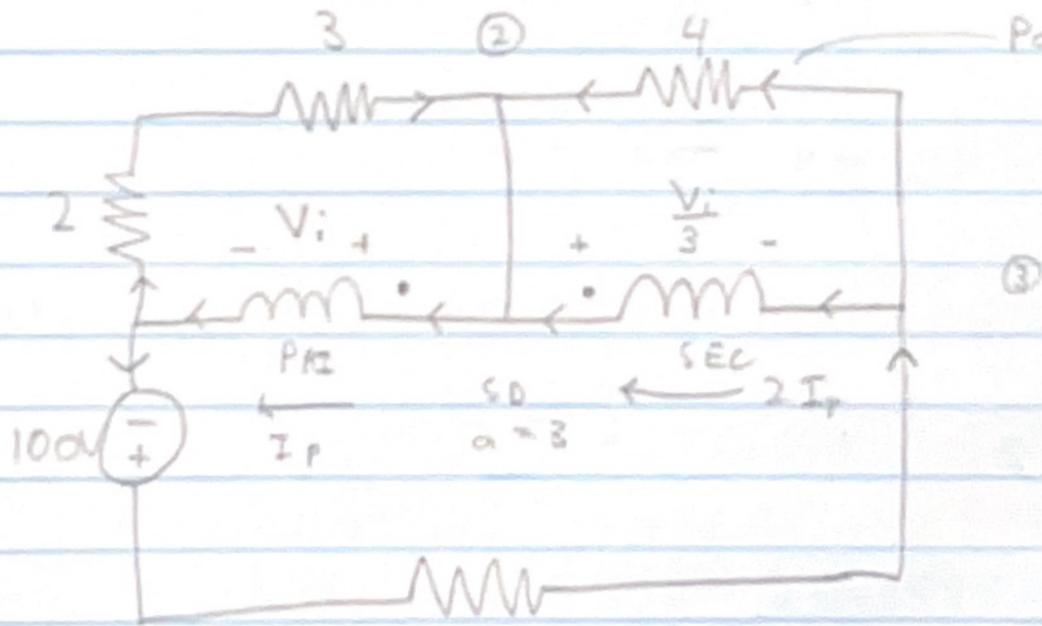
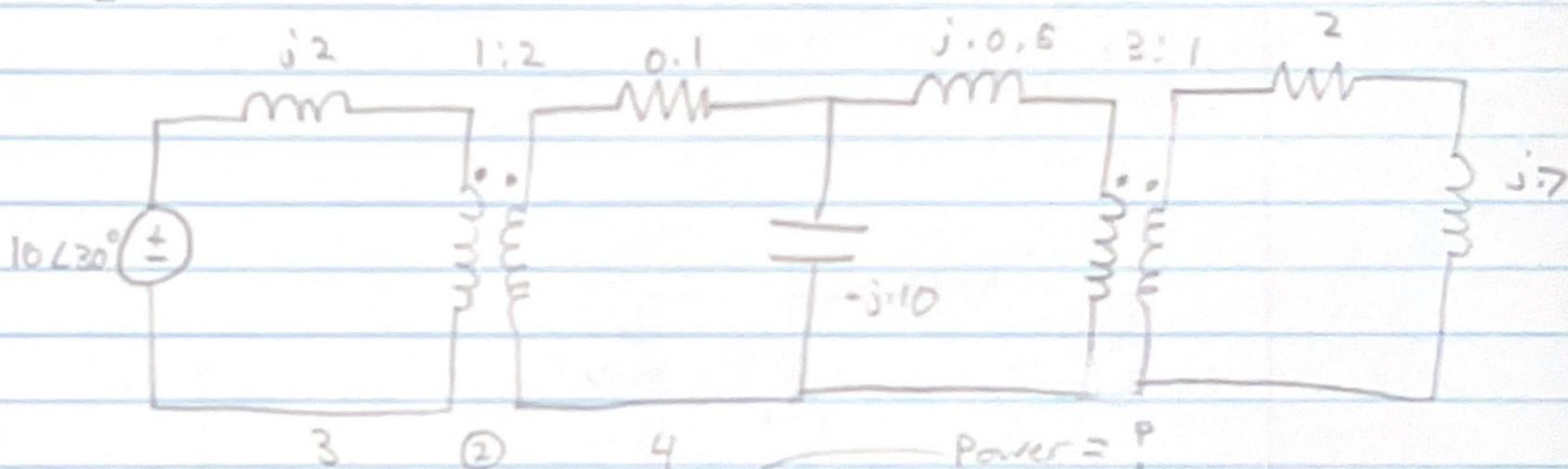




LV → HV :

- $V_s \cdot a$
- $I_s \div a$
- $Z \cdot a^2$

Tutorial



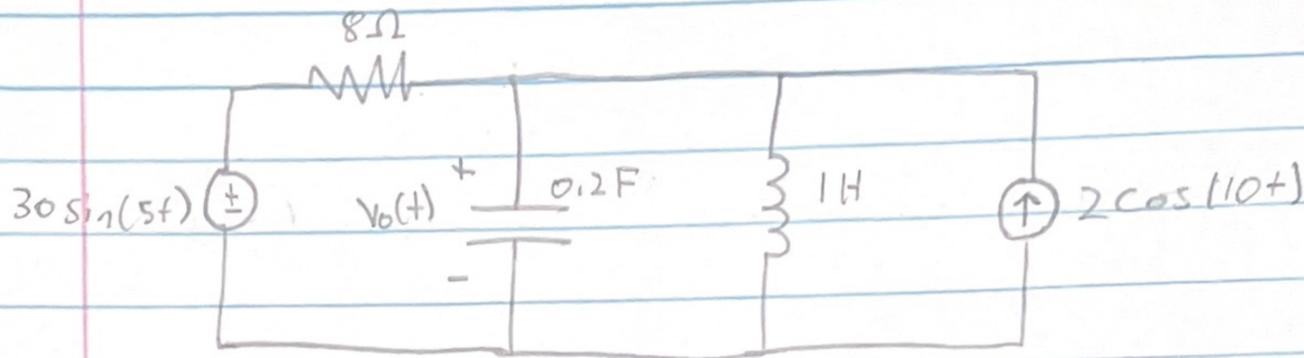
KCL1:  $I_p = \frac{0 - V_2}{5} + \frac{0 + 100 - V_1}{5}$

KCL2:  $\frac{V_1 + 100 - V_3}{5} = 2I_p + \frac{V_3 - V_2}{4}$

KCL3:  $\frac{V_1 - V_2}{5} + \frac{V_3 - V_2}{4} + 2I_p = I_p$

EVL:  $V_2 - V_1 = V_1$

SS 2.13

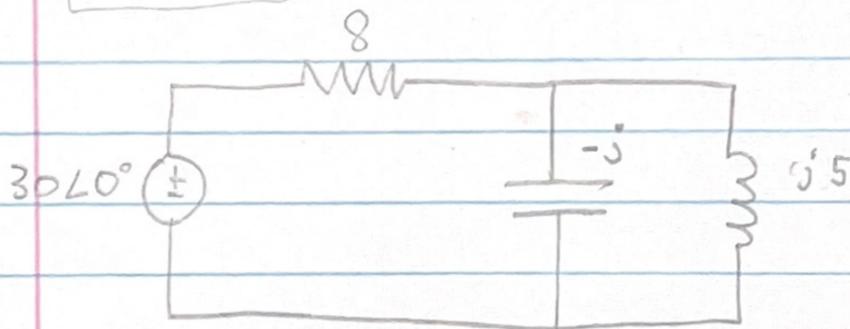


Superposition  $\rightarrow V_o = V_o' + V_o''$

$V_o'$  (voltage source)

$V_o''$  (current source)

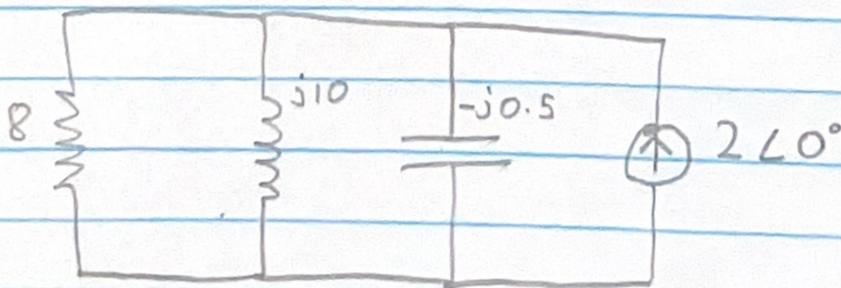
for  $V_o'$



$$V_o' = \frac{-j // j5}{8 + (-j // j5)} \cdot 30 = 4.631 \angle -81.12^\circ$$

$$v_o'(t) = 4.631 \sin(5t - 81.12^\circ) \text{ V}$$

for  $V_o''$   $2 \cos(10t) \rightarrow 2 \angle 0^\circ$



$$V_o'' = \frac{-j0.5}{8 // j10 + (-j0.5)} \cdot 2 \angle 0^\circ$$

$$= 1.051 \angle -86.24^\circ$$

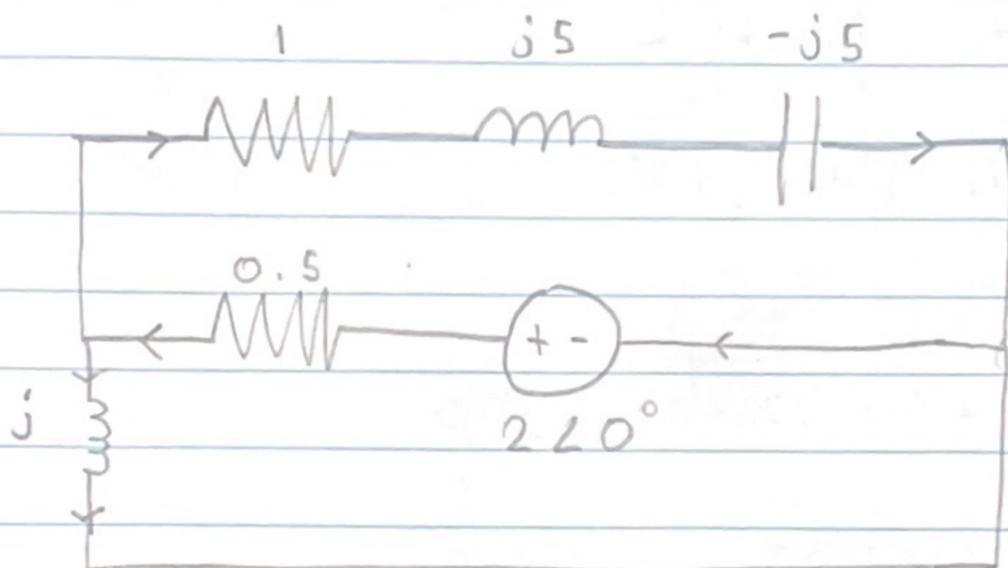
$$V_o''(t) = 1.051 \cos(10t - 86.24^\circ) V$$

Superposition

1.32

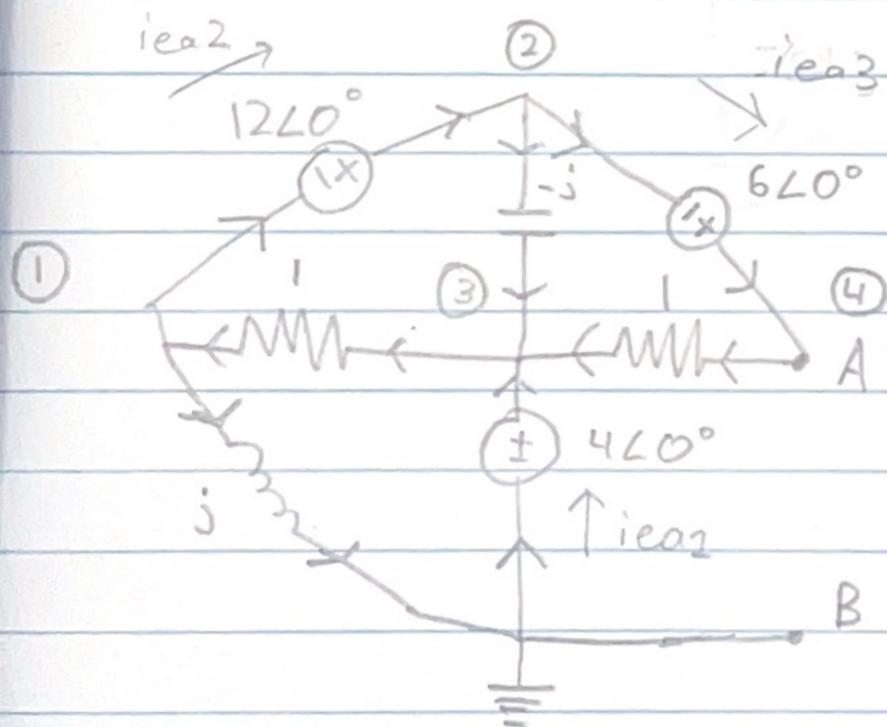
1.7

$$V_o(t) = 4.631 \sin(5t - 81.12^\circ) + 1.051 \cos(10t - 86.24^\circ)$$



REF

$$V_o(t) = 3.92 \cos(1000t - 71.31^\circ) V$$



$$\text{EVL1: } V_1 + 12 = V_2$$

$$\text{EVL2: } 0 + 4 = V_3$$

$$\text{EVL3: } V_2 + 6 = V_4$$

$$\text{KCL}_2: (V_3 - V_1) = \frac{V_1 + i_{ea2}}{j}$$

$$\text{KCL}_2: i_{ea2} = \frac{-(V_2 - V_3)}{-j} + i_{ea3}$$

$$\text{KCL}_3: (V_4 - V_3) + \frac{(V_2 - V_3)}{-j} + i_{ea1} = (V_3 - V_1)$$

$$\text{KCL}_4: i_{ea3} = (V_4 - V_3) \quad V_4 = 13 - j4$$

$$* \boxed{\text{TH2}} \quad \boxed{V_{TH} = 13 - j4}$$

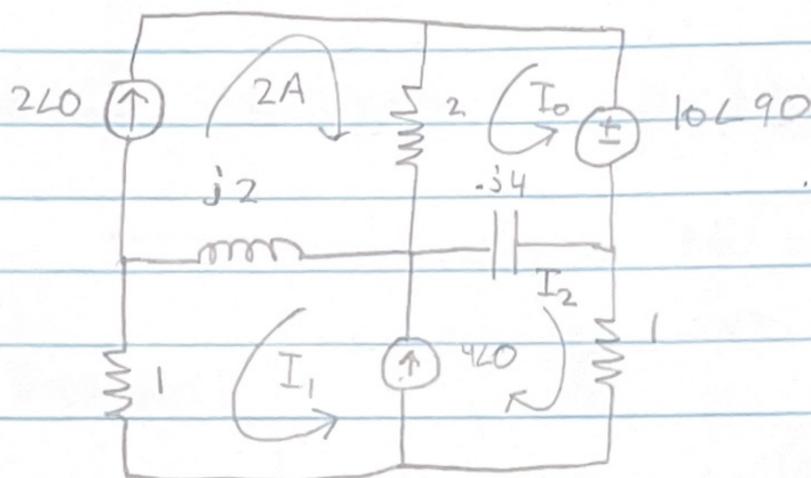
# Sinusoidal Steady State

2/4/19

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SS 3.6

Mesh Analysis:



$$\text{KVL}_1: (2 - j4)I_0 + (-j4)I_2 + 2 \cdot 2 = j \cdot 10$$

$$\text{(Bottom Rect) KVL}_2: -(-j4)I_0 + (1 + j2)I_1 - (1 - j4)I_2 + j2 \cdot 2 = 0$$

$$\text{KVL}_3: I_1 + I_2 = 4$$

Sol'n:

$$I_0 = 3.35 \angle 174.3^\circ \text{ A}$$

$$I_1 = 3.02 \angle -6.34^\circ \text{ A}$$

$$I_2 = 1.054 \angle 18.43^\circ \text{ A}$$

# AC Power

3/6/19

LEC

SS 4.2

"Power is the time rate of expending or absorbing energy"

Positive Power  $\rightarrow$  element is absorbing power

Negative Power  $\rightarrow$  element is supplying power

SS 4.3

$$P(t) = v(t)i(t) = V_m \cos(\omega t + \theta_v) I_m \cos(\omega t + \theta_i)$$

Trig Identity  $\leftarrow$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

SS 4.5

$$\frac{1}{T} \int_0^T P(t) dt = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

- ①  $P_{avg}$  is not time dependent (it is constant)
- ② When  $\theta_v = \theta_i \rightarrow$  it is a purely resistive load resulting in  $P = \frac{1}{2} V_m I_m$
- ③ When  $\theta_v - \theta_i = \pm 90^\circ \rightarrow P = 0$  this is a purely reactive load

# AC Power Analysis

3/11/19  
LEC

SS 5.2

Average Power:

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

SS

$V_{rms} I_{rms}$        $\cos(\theta_v - \theta_i)$   
 Apparent Power      Power Factor

SS 5.3

- Apparent power is  $S = V_{rms} I_{rms}$
- Power factor  $pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$
- Power factor angle =  $(\theta_v - \theta_i)$ , or the angle of  $Z_{eq}$

SS 5.4

$$pf = \cos(\theta_v - \theta_i)$$

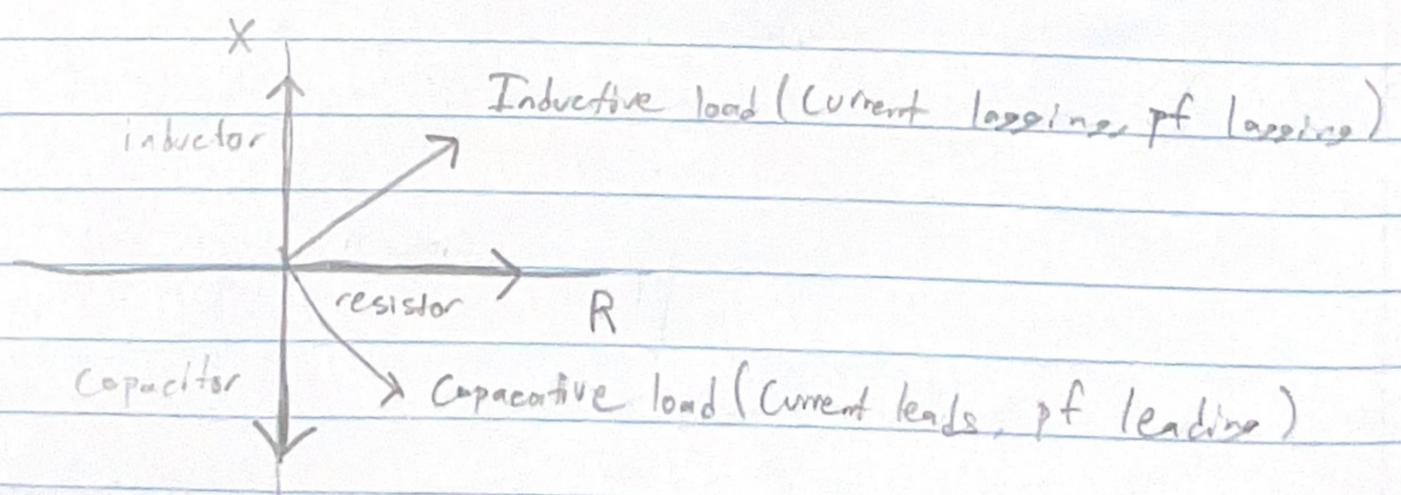
$$V = V_m \angle \theta_v$$

$$I = I_m \angle \theta_i$$

resistance

$$Z = \frac{V_m \angle \theta_v - \theta_i}{I_m} = R + jX$$

reactance



$$P_{avg} = V_{rms} I_{rms} \cdot (pf)$$

# Three Phase

3/18/19

LEC

SS 7.6

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ \quad \text{For abc sequence}$$

$$V_{cn} = V_p \angle -240^\circ \\ = V_p \angle 120^\circ$$

SS 7.9

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ$$

$$= V_p \angle 120^\circ$$

phase  
voltages

$$V_{ab} = V_{an} - V_{bn}$$

$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= V_p \left( 1 + \frac{1}{2} + j\frac{\sqrt{3}}{2} \right)$$

$$= \sqrt{3} V_p \angle 30^\circ$$

Line

Voltages

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = \sqrt{3} V_p \angle -210^\circ$$

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$$

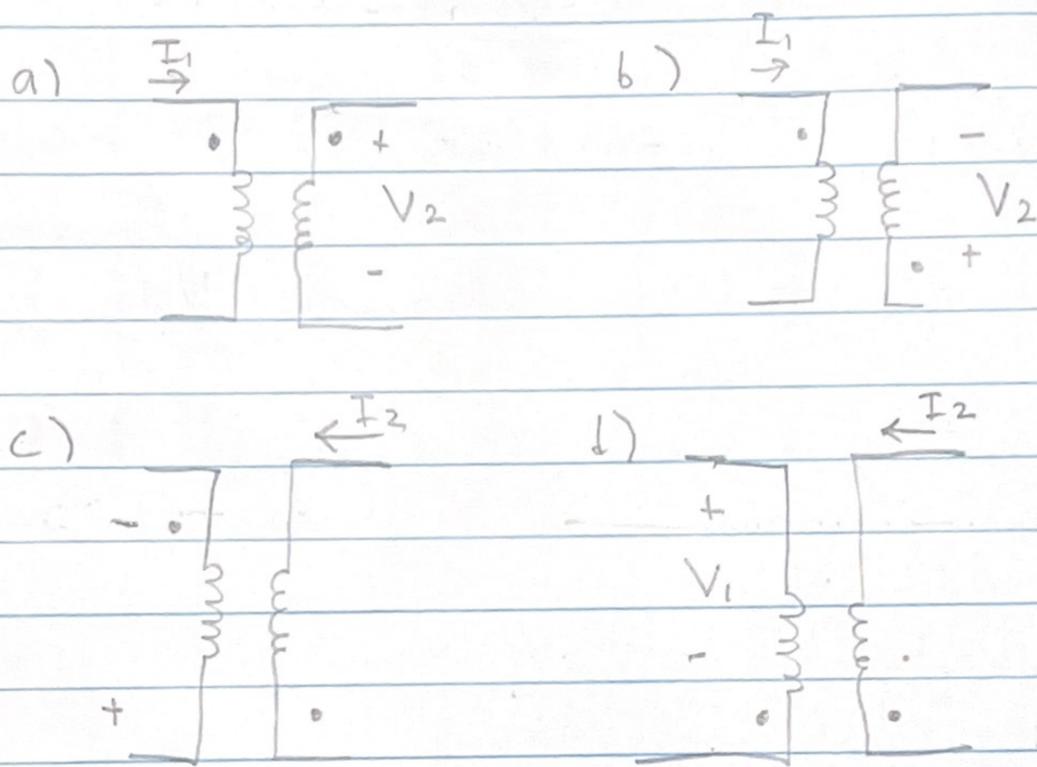
$$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|$$

$$= \sqrt{3} V_p$$

SS 8.5

- RHR
- Winding orientation affects mutual induction
- dot convention

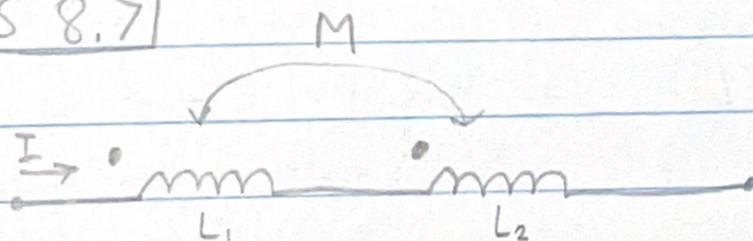
SS 8.6



"Self inductance induced voltage follows direction of current"

"Dot convention only applies to mutual induction"

SS 8.7



In Phasor Domain:

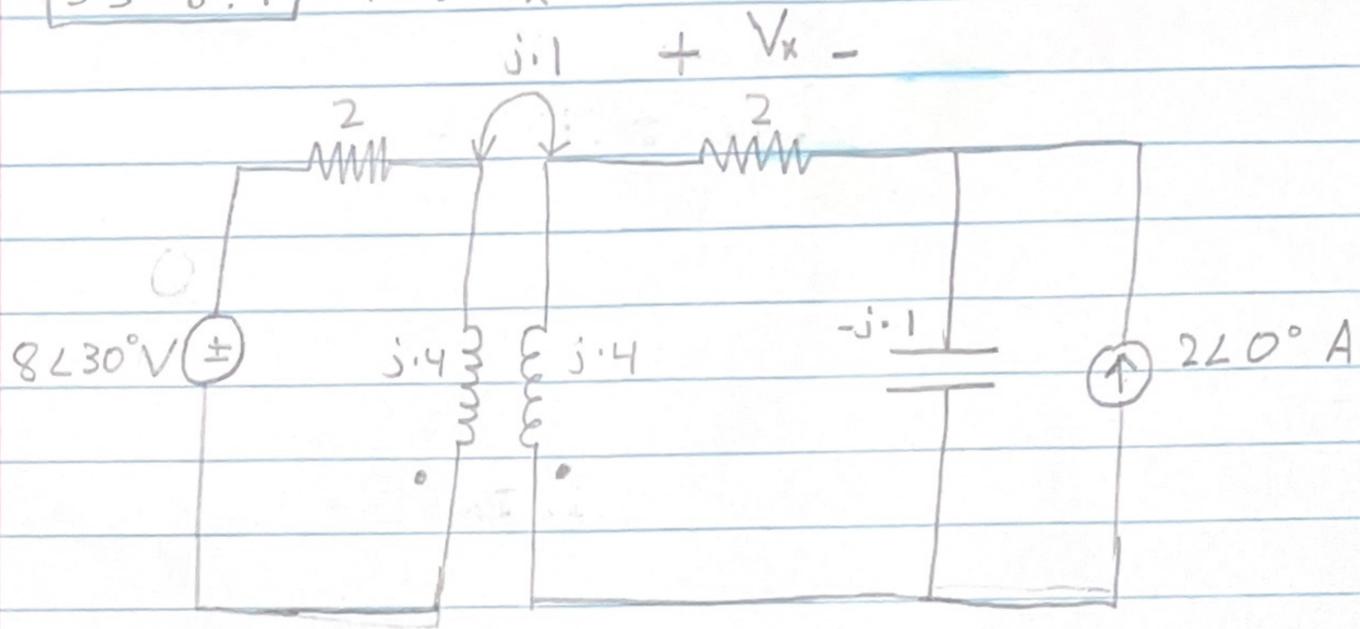
$$Z_L = j\omega L$$

$$V = j\omega L \cdot I \quad (\text{Self inductance})$$

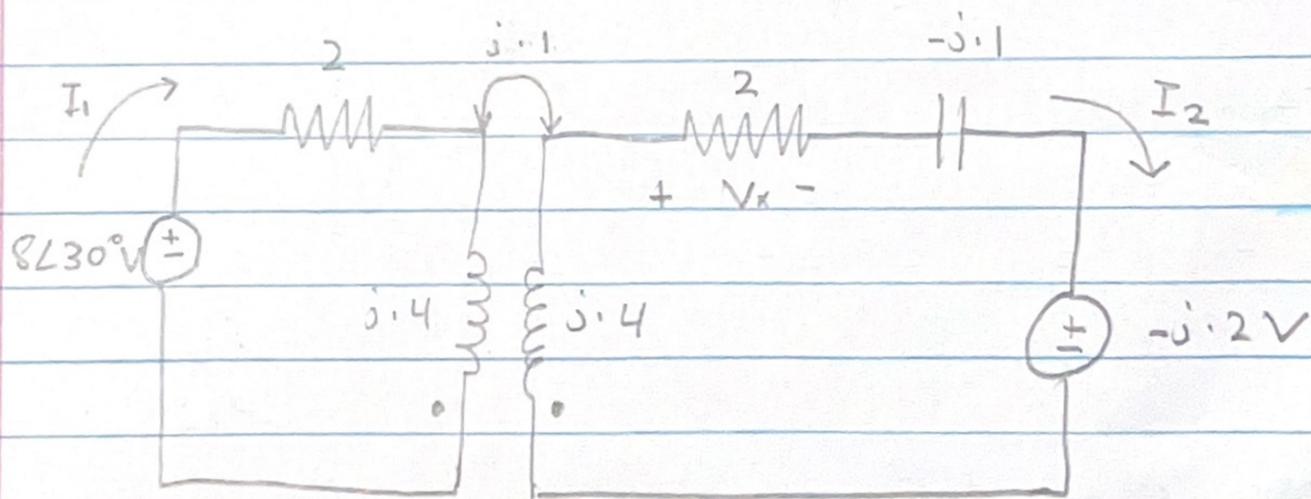
$$Z_M = j\omega M$$

$$V_M = j\omega M \cdot I \quad (\text{Mutual Inductance})$$

SS 8.9 Find  $V_x$



↓ Source Transformation



$$\text{KVL}_1: 8\angle 30^\circ = 2I_1 + j\cdot 4I_1 - j\cdot 1\cdot I_2$$

$$\text{KVL}_2: j\cdot 4\cdot I_2 + (2 - j\cdot 1)I_2 - j\cdot 2 - j\cdot 1\cdot I_1 = 0$$

$$\rightarrow I_2 = 1.037\angle 21.12^\circ \text{ A}$$

$$V_x = 2I_2 = 2.074\angle 21.12^\circ \text{ V}$$